9. V. S. Kosorygin, N. F. Polyakov, L. A. Rumyantseva, and E. Ya. Epik, "Certain aspects of the use of a hot-wire anemometer to study the structure of a turbulent boundary layer," in: Physical Gasdynamics [in Russian], Vol. 6, ITPM Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1976), pp. 217-218.
10. V. T. Kiril'tsev, V. P. Motulevich, and E. D. Sergievskii, "Effect of mean flow temperature on the readings of a hot-wire anemometer," Inzh.-Fiz. Zh., 42, No. 4, 622-627 (1982).
11. V. T. Kiril'tsev, V. P. Motulevich, and E. D. Sergievskii, "Measurements with a hot-wire anemometer in highly turbulent boundary layers," Promyshlennaya Teplotekhnika, 4, No. 5, 96-101 (1982).
12. E. U. Repik and V. S. Ponomareva, "Study of the effect of proximity of the wall on the readings of a hot-wire anemometer in a turbulent boundary layer," Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, No. 13, 45-52 (1969).
13. G. Schlichting, Boundary Layer Theory, McGraw-Hill (1960).
14. E. M. Khabakhpasheva and E. S. Mikhailova, "Study of turbulence in the flow of water in a channel with parallel walls," in: Experimental Study of the Structure of Boundary-Layer Turbulence and the Viscous Sublayer [in Russian], ITF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1976), pp. 33-57.
15. A. A. Klimov, E. D. Sergievskii, and L. S. Yanovskii, "Turbulent boundary layer with directional injection through a permeable plate," Nauchn. Tr. Mosk. Lesotekh. Inst., 130, 36-43 (1981).
16. R. L. Simpson, W. M. Kays, and R. J. Moffat, "The turbulent boundary layer on a porous plate: an experimental study of the fluid dynamics with injection and suction," Report N HMT-2, Stanford, California (1967).
17. B. P. Mironov and V. N. Mamonov, "Effect of external turbulence on flow hydrodynamics in a turbulent boundary layer with intensive uniform injection," in: Turbulent Transport with Injection on a Surface [in Russian], ITF Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1980), pp. 7-21.

LOSS OF A PASSIVE IMPURITY IN A TURBULENT VORTEX RING

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UDC 532.72

A diffusion boundary-layer approximation is used to obtain an analytic solution to the problem of the loss of a passive impurity by a turbulent vortex ring.

Turbulent vortex rings have long interested many investigators due to the relative ease of obtaining them, the transfer of impurities and their travel over long distances, the long-term stability of the rings, etc. Thus, here we study the possibility of making practical use of vortex rings to remove smoke and harmful gases at industrial plants, to remove contaminants from the walls of various types of containers, etc.

There are many methods of organizing vortex rings [1]: surface explosion of a large quantity of explosive [2], injection of a liquid of one density into a liquid medium of a different density [3, 4], etc. Henceforth, for the sake of definiteness we will have in mind a turbulent vortex ring (TVR) obtained in a container filled with smoke (a Wood box [5]) and having an explosive charge on its bottom. However, the theory proposed here is applicable for other methods of producing vortex rings.

There are two types of TVR's created by a vortex generator: toroidal [3, 6, 7], which loses nearly all of the impurity it transports during its motion; ellipsoidal [8]. In contrast to the former, the latter are formed by preliminary agitation of the flow, such as by the installation of a metal grid in the working part of the generator nozzle. In this case, small quantities of impurity are lost in the wake. The resulting vortex consists of a core — a toroidal vortex — surrounded by a moving shell having the form in the direction of motion of an oblate ellipsoid of revolution. Continuity of the velocity field outside and inside the vortex is maintained (similar to the motion of a drop in a liquid). The main losses of passive

Department of Theoretical Problems, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 50, No. 1, pp. 31-39, January, 1986. Original article submitted June 20, 1984.

 0022-0841/86/5001-0022$12.50

impurity, i.e., impurities not affecting the motion of the TVR, generally occur in this instance in a short initial time interval starting from the moment t_0 of formation of the vortex and the beginning of self-similar motion. These losses are small compared to the total (maximum) quantity of impurity transported by the vortex, as was confirmed in an experiment [8].

A problem was formulated in [9, 10] for finding the velocity field of a TVR and the concentration of passive impurity transported by it on the basis of averaged equations describing turbulent flow in an incompressible medium (Helmholtz equations) and convective diffusion in cylindrical coordinates (r, z) connected with the center of the outlet hole of a vortex generator. Here, the turbulent character of motion was accounted for by introducing the eddy viscosity coefficient $\nu_*(t)$. As follows from experiments, over a significant length of vortex travel, this coefficient is many times greater than the molecular viscosity ν. As a result, it was established that these processes are self-similar, and the initial equations in self-similar variables were transformed into steady equations. Approximate relations were found to describe the time dependence of the path travelled by the vortex:

$$L(t) = R_0\alpha^{-1}[(1 + 4\alpha V_0 R_0^{-1} t)^{1/4} - 1], \tag{1}$$

and the time dependence of the radius of the vortex

$$R(t) = R_0 + \alpha L(t).$$

Chosen here as the initial moment was the beginning of self-similar motion, which delimits the interval of time t_0 of the motion of the TVR from the generator nozzle to the establishment of similitude (here, the vortex travels a certain distance — usually 4-5 diameters of the nozzle aperture):

$$t_0 = \frac{1}{4}\frac{R_0}{\alpha V_0}, \tag{2}$$

α is a constant determined by comparison of the calculated results with experimental results and thus found to be a small quantity on the order of 10^{-2}-10^{-3}; it should also be noted that the radius of the vortex and the distance it travels are the values $r = R(t)$ and $z = L(t)$ at which the vorticity $\Omega(t, r, z)$ has a maximum with a fixed t.

However, given this formulation of the problem, it is very complicated to find the distribution of velocity and vorticity in the TVR, as well as the concentration of impurity transported by it. This is because the cylindrical coordinate poorly reflect the geometric features of the structure of the vortex.

Henceforth examining ellipsoidal TVR's formed by preliminary agitation of the flow, it can be concluded on the basis of the general similitude of the process and the experimental results in [1, 8, 11] that the form of the vortex shell changes during its motion over time in a manner geometrically similar to the original vortex, i.e., it has the form of an oblate ellipsoid of revolution expanding in the direction of motion (Fig. 1).

Assuming that the center of the ellipsoid coincides with the center of the vortex, we use $a(t)$, $b(t)$, and $s(t)$ to designate running values of the major semiaxis, minor semiaxis, and focal length of the ellipsoidal shell. Then by virtue of the similitude of the process, these quantities will be linked with their initial values $a(t_0) = a_0$, $b(t_0) = b_0$, $s(t_0) = s_0$ by the relations

$$\begin{gathered} a(t) = (P_0 t)^{1/4} a_*, \quad a_* = a_0 (P_0 t_0)^{-1/4}, \\ b(t) = (P_0 t)^{1/4} b_*, \quad b_* = b_0 (P_0 t_0)^{-1/4}, \\ s(t) = \sqrt{a^2(t) - b^2(t)} = (P_0 t)^{1/4} s_*, \quad s_* = s_0 (P_0 t_0)^{-1/4}, \end{gathered} \tag{3}$$

where P_0 is the momentum of the vortex. Based on the conservation law [9, 10] in an infinite viscous incompressible liquid, P_0 is a constant (independent of time) under the conditions that vorticity decays sufficiently rapidly at infinity. The value of P_0

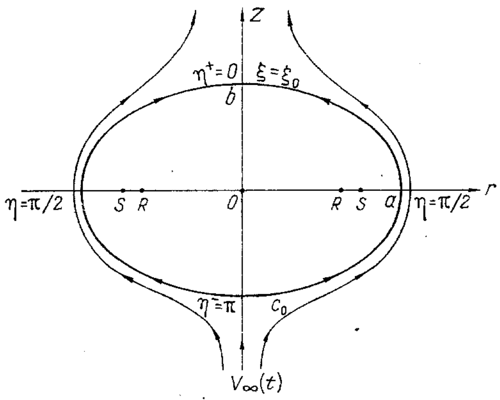

Fig. 1. Ellipsoidal vortex.

$$P_0 = \frac{1}{4} q \alpha^{1/5} V_0 R_0^3,$$
$$q = \left\{ \frac{1}{16} \pi^{-3/2} \exp\left(-\frac{1}{2}\right) \left[1 - \sqrt{\pi}\, 2^{-3/2} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right] \right\}^{-4/5} \approx 84{,}633. \quad (4)$$

Equations (3) give the self-similar law of expansion of the shell of an ellipsoidal TVR. Knowing a_0 and b_0, it is possible to use (3) to find the form of the shell at any moment of time, i.e., to find the quantities a(t) and b(t). The values of a_0 and b_0 are determined either by experiment or by solving the hydrodynamic problem.

We will introduce a system of coordinates, connected with the center of the vortex, for an oblate ellipsoid of revolution (ξ, η, φ) using the formulas in [12]: ($0 \leqslant \xi < \infty$, $0 \leqslant \eta \leqslant \pi$, $0 \leqslant \varphi \leqslant 2\pi$):

$$Z = s(t) \operatorname{sh} \xi \cos \eta, \quad r = s(t) \operatorname{ch} \xi \sin \eta, \quad (5)$$

where (Z, r) is a cylindrical system of coordinates with its origin at the center of the vortex. Here

$$Z = z - L(t) - z(t_0);$$
$$z(t_0) = P_0^{1/4} x_0(\lambda)\, t_0^{1/4}, \; x_0(\lambda) = \frac{1}{8} \pi^{-3/2} \exp\left(-\frac{1}{2}\right) \left[1 - \sqrt{\pi}\, 2^{-3/2} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right] \lambda^{-2},$$
$$\lambda = \left\{ \frac{1}{16} \pi^{-3/2} \exp\left(-\frac{1}{2}\right) \left[1 - \sqrt{\pi}\, 2^{-3/2} \operatorname{erf}\left(\frac{1}{\sqrt{2}}\right) \right] \alpha \right\}^{2/5} \approx 0.1087 \alpha^{2/5}, \quad (6)$$

where (z, r) is a cylindrical system of coordinates referred to the center of the outlet hole of the vortex generator (the coordinate r remains unchanged), while $z(t_0)$ is the initial distance (from the hole of the generator to the formation of the vortex and the beginning of self-similar motion) [9, 10]. It should be noted that for this coordinate system, the coordinate surfaces ξ = const are oblate ellipsoids of revolution which are confocal for each moment of time (Fig. 1). Here, the equation of the surface the ellipsoidal shell of the vortex will have the form $\xi = \xi_0$. Meanwhile, as it is easy to see from (3) and (5):

$$\xi_0 = \operatorname{arth}[b(t)/a(t)] = \operatorname{arth}(b_0/a_0).$$

Thus, the boundary surface of the TVR, exposed to an incoming flow with the velocity $V_\infty(t) = dL/dt$, has the fixed shape of an ellipsoid of revolution which is oblate in the direction of motion in dimensionless coordinates. The forward critical inflow point and the rear critical outflow point on the surface of the TVR shell are respectively ($\xi=\xi_0$, $\eta^-=\pi$) and ($\xi=\xi_0$, $\eta^+=0$).

Using an axisymmetric formulation, we will examine an external problem on convective diffusion of a passive impurity into the environment from the surface of an independently moving ellipsoidal TVR having the form of an ellipsoid of revolution oblate in the direction of motion. It should be noted that the region of the core of the vortex requires special additional studies due to features of the effect of vorticity in this region on mass transfer inside the elliposidal boundary of the vortex [13, 14].

In the case of turbulent motion of the fluid, the transport of impurities in the absence of boundaries can be described by introducing a special eddy diffusion coefficient D_*. As the value of the eddy viscosity coefficient ν_*, the value of this coefficient is determined by the characteristic scale of motion, i.e., by the size and velocity of the vortex. It is known from experiments with turbulent jets [15] that to within a factor on the order of unity ($\gamma \approx 1.2$-1.3), D_* coincides with the eddy viscosity coefficient [1, 9, 10]:

$$D_*(t) = \gamma \nu_*(t), \ \nu_*(t) = \lambda P_0^{1/2} t^{-1/2}. \tag{7}$$

Then the equation of convective diffusion describing the distribution of the concentration C of the passive impurity during the motion of the TVR has the form [1, 9, 10]:

$$\partial C/\partial t + (\mathbf{V}\cdot\nabla) C = \gamma \nu_*(t)\, \Delta C. \tag{8}$$

Here, it was considered that on the section of self-similar motion of the vortex being examined, the molecular diffusion coefficient D can be ignored compared to the eddy diffusion coefficient D_*. The velocity field **V** is found from solution of the hydrodynamic problem, while by the concentration C we mean the quantity of impurity per unit volume of the medium.

In the system of coordinates (ξ, η), the process of loss of the passive impurity by the vortex will be described by equation of convective diffusion (8) in dimensionless form ($0<\varepsilon=\gamma\lambda\ll 1$):

$$\begin{gathered}
-\frac{3}{4}c + H\left(U_\xi \frac{\partial c}{\partial \xi} + U_\eta \frac{\partial c}{\partial \eta}\right) = \varepsilon H^2 H_\varphi \left[\frac{\partial}{\partial \xi}\left(\frac{1}{H_\varphi}\frac{\partial c}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(\frac{1}{H_\varphi}\frac{\partial c}{\partial \eta}\right)\right], \\
H = s_*^{-1}(\operatorname{ch}^2\xi - \sin^2\eta)^{-1/2}, \ H_\varphi = s_*^{-1}(\operatorname{ch}\xi \sin\eta)^{-1}, \\
U_\xi(\xi, \eta) = V_\xi(t, \xi, \eta) P_0^{-1/4} t^{3/4}, \ U_\eta(\xi, \eta) = V_\eta(t, \xi, \eta) P_0^{-1/4} t^{3/4}, \\
U_\xi = -HH_\varphi \frac{\partial \psi}{\partial \eta}, \ U_\eta = HH_\varphi \frac{\partial \psi}{\partial \xi}, \ \psi(\xi, \eta) = \Psi(t, \xi, \eta) P_0^{-3/4} t^{1/4}, \\
c = c(\xi, \eta) = Q_0^{-1} (P_0 t)^{3/4} C(t, \xi, \eta);
\end{gathered} \tag{9}$$

it being considered here that the impurity is dispersed by the vortex into an environment uncontaminated by suspended particles, so that

$$C_\infty = c_\infty = 0, \tag{10}$$

while the concentration of impurity on the moving ellipsoidal surface of the TVR during the entire time of the self-similar process changes according to the law (Fig. 1):

$$C_0 = Q_0 (P_0 t)^{-3/4} c_0, \ (c_0 = \text{const}), \tag{11}$$

which means that at the moment of time t_0 of the beginning of self-similar motion, all of the impurity is uniformly distributed in the vortex with a concentration $C_0(t_0) = Q_0(P_0 t_0)^{-3/4} c_0$. During subsequent motion impurity is removed into the environment from the shell of the TVR in accordance with self-similar law (11).

Since the small parameter $\varepsilon = \gamma\lambda$ is a multiplier in Eq. (9) with the higher derivatives, we have the classical problem of a diffusion boundary layer adjacent to the surface of a vortex within which diffusion transport of the substance along this surface (along η) can be ignored compared to transport in the direction normal to it (along ξ) [16, 17]. With allowance for this, we write the equation of convective diffusion (9) in a diffusion boundary layer approximation

$$-\frac{3}{4}c - H^2 H_\varphi \left(\frac{\partial \psi}{\partial \eta}\frac{\partial c}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial c}{\partial \eta}\right) = \varepsilon H^2 H_\varphi \frac{\partial}{\partial \xi}\left(\frac{1}{H_\varphi}\frac{\partial c}{\partial \xi}\right) \tag{12}$$

and the corresponding boundary conditions

$$c(\xi = \xi_0, \eta) = c_0, \ c(\xi \to \infty, \eta) \to 0, \ c(\xi, \eta = \eta^- = \pi) = 0, \tag{13}$$

the first two of which are the conditions on the surface of the vortex shell and at infinity. The last condition expresses the absence of suspended particles in the hydrodynamic flow on the inflow path $\eta^- = \pi$ [17]. The solution of boundary-value problem (12-13) gives the distribution of the dimensionless concentration of the impurity lost by the vortex to the uncontaminated environment during its motion.

To obtain this, we replace the variables (ξ, η) with the von Mises variables (ψ, η) [18]. As a result, Eq. (12) takes the form

$$-\frac{3}{4} c + H_0 U_{0\eta} \frac{\partial c}{\partial \eta} = \varepsilon H_{0\varphi}^{-2} U_{0\eta}^2 \frac{\partial^2 c}{\partial \psi^2}, \tag{14}$$

where the subscript 0 henceforth indicates that the corresponding quantities depend on $\xi = \xi_0$, η, i.e., are taken on the surface of the vortex shell.

Let us change over from the variables (ψ, η) to the variables (ψ, σ), where

$$\begin{gathered} \sigma = -\varepsilon s_* (1 + \xi_0^{*2})^{1/2} \int_{-1}^{\eta^*} (H_0 H_{0\varphi})^{-1} U_{0\lambda} d\lambda; \ H_0 = s_*^{-1} (\xi_0^{*2} + \eta^{*2})^{-1/2}, \\ H_{0\varphi} = s_*^{-1} [(1 + \xi_0^{*2})(1 - \eta^{*2})]^{-1/2}, \ \eta^* = \cos\eta, \\ \xi^* = \operatorname{sh}\xi, \ (\xi_0^* = \operatorname{sh}\xi_0). \end{gathered} \tag{15}$$

Then, introducing the new function

$$\begin{gathered} u = u(\psi, \eta^*) = -c \exp[-\varphi(\eta^*)]; \\ \varphi(\eta^*) = -\frac{3}{4} \int_{\tilde{\eta}^*}^{\eta^*} H_0^{-1}(\lambda) U_{0\lambda}^{-1} (1 - \lambda^2)^{-1/2} d\lambda, \end{gathered} \tag{16}$$

where $\tilde{\eta}^*$ is a certain fixed value of η^*, we find that Eq. (14) and boundary conditions (13) take the following forms:

$$\partial u / \partial \sigma = \partial^2 u / \partial \psi^2; \tag{17}$$

$$\psi = 0, \ u = -c_0 \exp\{-\varphi[\eta^*(\sigma)]\}; \ \psi \to \infty, \ u \to 0; \ \sigma = 0, \ u = 0. \tag{18}$$

The solution of boundary-value problem (17)-(18) is the function [19]

$$u(\psi, \sigma) = -\frac{c_0}{2\sqrt{\pi}} \psi \int_0^{\sigma} (\sigma - z)^{-3/2} \exp\left\{-\frac{\psi^2}{4(\sigma - z)} - \varphi[\eta^*(z)]\right\} dz.$$

Using Eq. (16) to change over to the concentration, we finally obtain the solution of the problem in general form

$$c = \frac{c_0}{2\sqrt{\pi}} \psi \exp[\varphi(\eta^*)] \int_0^{\sigma} (\sigma - z)^{-3/2} \exp\left\{-\frac{\psi^2}{4(\sigma - z)} - \varphi[\eta^*(z)]\right\} dz,$$

where σ and $\varphi(\eta^*)$ are expressed by Eqs. (15) and (16) [16].

The local $\tilde{j}$ and total $\tilde{I}$ diffusion flows of the passive impurity from the surface of the vortex shell, representing the rates of removal of impurity from a unit surface and from the entire surface of the shell, respectively, are determined as follows at a given moment of time [17]

$$\begin{gathered} \tilde{j} = D_* \left(\frac{\partial C}{\partial n}\right)_\Sigma = D_* \left(h_\xi \frac{\partial C}{\partial \xi}\right)_{\xi = \xi_0}, \ \tilde{I} = \int_\Sigma \tilde{j} d\Sigma \\ (h_\xi = (P_0 t)^{-1/4} H = s^{-1}(t) (\operatorname{ch}^2\xi - \sin^2\eta)^{-1/2}), \end{gathered} \tag{19}$$

where $\mathbf{n}$ is the vector of a unit normal to the surface $\{\Sigma : \xi = \xi_0\}$ of the vortex shell.

Introducing the dimensionless diffusion flows j and I by means of the formulas

$$\tilde{j} = jQ_0P_0^{-1/2}\, t^{-3/2},\quad \tilde{I} = IQ_0t^{-1}$$

and considering (19), we obtain the following expressions for these flows

$$j = \gamma\lambda H_0\left(\frac{\partial c}{\partial \xi}\right)_{\xi=\xi_0},\quad I = 2\pi\int\limits_0^\pi jH_0^{-1}H_{0\varphi}^{-1}d\eta = 2\pi\gamma\lambda\int\limits_0^\pi H_{0\varphi}^{-1}\left(\frac{\partial c}{\partial \xi}\right)_{\xi=\xi_0}d\eta. \tag{20}$$

The total amount of impurity $\tilde{I}_0$ lost by the TVR during its motion over the period of time τ $(t_0 \leqslant \tau \leqslant t_1)$ is represented as

$$\tilde{I}_0(\tau) = \int\limits_{t_0}^{\tau}\tilde{I}dt = \int\limits_{t_0}^{\tau}IQ_0t^{-1}dt = Q_0I\ln(\tau/t_0). \tag{21}$$

Using (9), (15), and (20), we write the dimensionless total diffusion flow in the form

$$I = 2\pi\varepsilon s_*(1+\xi_0^{*2})\int\limits_{-1}^{1}\left(\frac{\partial c}{\partial \xi^*}\right)_{\xi^*=\xi_0^*}d\eta^* = 2\pi\int\limits_{\sigma(1)}^{0}\left(\frac{\partial c}{\partial \psi}\right)_{\psi=0}d\sigma. \tag{22}$$

Since $0<\varepsilon=\gamma\lambda\ll 1$ and since the integrand functions in (15) are continuous and bounded, we expand $(\partial c/\partial\psi)_{\psi=0}$ into a power series in the small parameter σ and integrate term by term in accordance with (22). As a result, leaving the dominant terms of the expansion, we obtain an approximate expression for the total diffusion flow:

$$I = 2\pi c_0\sqrt{|\sigma(1)|} = 2\pi\sqrt{\varepsilon}\, c_0\sqrt{|\sigma_*(1)|},$$
$$\sigma_*(1) = -s_*(1+\xi_0^{*2})^{1/2}\int\limits_{-1}^{1}(H_0H_{0\varphi})^{-1}U_{0\lambda}d\lambda. \tag{23}$$

Thus, to determine the diffusion flow it is necessary to know the flow field on the boundary of the TVR shell $\xi = \xi_0$, i.e., $U_{0\eta}$. Since the turbulent flow is concentrated mainly within the ellipsoidal boundary of the vortex and turbulence decays rapidly outside this boundary going away from it [1-4, 7-11], the results in [20] will be used to approximately determine the flow field outside the TVR shell. This study examined the velocity distribution in steady laminar flow about a gas bubble having the form of an ellipsoid of revolution oblate in the direction of the incoming flow. The bubble was in a low-viscosity fluid (here, the Reynolds number Re, taken on the minor semiaxis b of the ellipsoid, was assumed to be large).

Considering that the TVR moves at a variable velocity, we subdivide the interval of time of self-similar motion into N small segments. Within each of these segments, the rate of flow for the vortex can be approximately taken as a constant $V_\infty(t_i)$:

$$U_{i\infty}^* = V_\infty(t_i)P_0^{-1/4}\, t_i^{3/4}.$$

The thus-introduced piecewise-constant velocity of the vortex allows us to examine the problem as being quasisteady, and on each small segment of the time of motion of the vortex we use the results in [20], according to which $(\mathrm{Re}=V_\infty(t_i)b(t_i)/\nu\gg 1)$:

$$U_{0\eta^*} = U_{\eta^*}(\xi^*=\xi_0^*,\ \eta^*) = U_{0\eta^*}^{(1)} + \mathrm{Re}^{-1/2}U_{0\eta^*}^{(2)},\quad U_{0\eta^*}^{(1)} = U_\infty^*\frac{\chi(\xi_0^*)}{1+\xi_0^{*2}}\left(\frac{1-\eta^{*2}}{\xi_0^{*2}+\eta^{*2}}\right)^{1/2}, \tag{24}$$

$$U_{0\eta^*}^{(2)} = U_\infty^*\left[\frac{2}{3}\,\frac{\chi(\xi_0^*)}{\xi_0^*}\,\frac{\xi_0^{*2}+\eta^{*2}}{1-\eta^{*2}}\right]^{1/2}\Lambda(\xi_0^*,\ \eta^*),$$

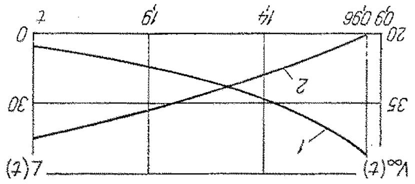

Fig. 2

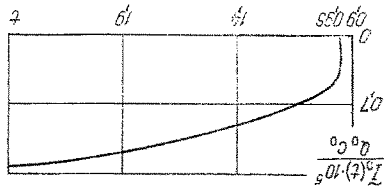

Fig. 3

Fig. 2. The function $Q_0^{-1} c_0^{-1} \tilde{I}_0(t) \cdot 10^5$. t, sec.

Fig. 3. The function $V_\infty(t)$ (m/sec) and L(t) (m).

$$\chi(\xi_0^*) = \left[\operatorname{arcctg} \xi_0^* - \frac{\xi_0^*}{1+\xi_0^{*2}}\right]^{-1},$$

$$\Lambda(\xi_0^*, \eta^*) = -\frac{2}{\sqrt{\pi}} \int_{-1}^{\eta^*} \frac{\xi_0^{*2}(1-x)^2}{(\xi_0^{*2}+x^2)^2} \frac{dx}{\sqrt{X(\eta^*)-X(x)}}, \tag{24}$$

$$X(x) = \frac{2}{9}(x+1)^2(2-x).$$

Here and in subsequent calculations, the subscript i is omitted for the sake of clarity.

Inserting this expression for velocity into (23), we find that the total diffusion flow is

$$I = 2\pi c_0 \sqrt{\varepsilon} \sqrt{|\sigma_*(1)|}, \quad \sigma_*(1) = \sigma_*^{(1)}(1) + \mathrm{Re}^{-1/2} \sigma_*^{(2)}(1),$$

$$\sigma_*^{(1)}(1) = -4/3 s_*^3 U_\infty^* \chi(\xi_0^*),$$

$$\sigma_*^{(2)}(1) = 2\sqrt{\frac{3}{\pi}} U_\infty^* s_*^3 \sqrt{\chi(\xi_0^*)}\, \xi_0^{*3/2} (1+\xi_0^{*2}) \int_{-1}^{1} (\xi_0^{*2} + \eta^{*2}) \Lambda_*(\xi_0^*, \eta^*)\, d\eta^*, \tag{25}$$

$$\Lambda_*(\xi_0^*, \eta^*) = -\frac{1}{3}\sqrt{\frac{\pi}{2}}\, \xi_2^{*-2} \Lambda(\xi_0^*, \eta^*),$$

$$(0 < \varepsilon = \gamma\lambda \ll 1, \; 0 < \mathrm{Re}^{-1/2} \ll 1).$$

To make concrete use of the formulas, we will examine the problem of the loss of a passive impurity by a TVR obtained experimentally by the explosion of a detonating fuse on the bottom of a cylindrical tank. Here, the initial parameters of the vortex propagating into the atmosphere were as follows: $\mathrm{Re}_0 = V_0 b_0/\nu = 1.65 \cdot 10^6$, $R_0 = 55$ cm, $V_0 = 4570$ cm/sec, $\nu = 0.152$ cm^2/sec, $b_0/a_0 = b(t)/a(t) = 0.502$, $s(t)/a(t) = R_0/a_0 = 0.865$, $a_0 = 63.61$ cm, $b_0 = 32$ cm, $\xi_0 = 0.553$, $\xi_0^* = 0.581$, $\alpha = 3.13 \cdot 10^{-3}$.

Using Eqs. (2-4), (6), and (24), we find: $t_0 = 0.961$ sec, $P_0 = 5.077 \cdot 10^9$ cm^4/sec, $\lambda = 0.0108$, $s_* = 0.208$, $\chi(\xi_0^*) = 1.64$, $\varepsilon = \gamma\lambda = 1.25$ $\lambda = 0.0135$. Taking these values into account and numerically integrating the double integral in the expression for $\sigma_*^{(2)}$ (25), (24), we obtain:

$$\sigma_*^{(1)}(1) = -0.327, \quad \sigma_*^{(2)}(1) = 1.612.$$

From this we arrive at the relation for the dimensionless diffusion flow (25) of the passive impurity from the surface of the vortex. This relation is valid for each small segment of time of self-similar motion, in which the velocity of the vortex is a constant:

$$I = 2\pi c_0 \sqrt{\varepsilon} \sqrt{|\sigma_*(1)|} = 0.731 c_0 \sqrt{0.327 - 1.612\, \mathrm{Re}^{-1/2}},$$

$$\mathrm{Re} = V_\infty(t_i)\, b(t_i)\, \nu^{-1}.$$

Proceeding on the basis of (21), we have the following equations to express the total amount of passive impurity $\tilde{I}_0$ lost by the TVR during the time of experimental observation $\delta t = t_N - t_0 \approx 1.45$ sec ($t_N \approx 2.41$ sec, $t_0 \approx 0.96$ sec, which is broken down into N small segments $\delta_i = t_i - t_{i-1}$

$$\tilde{I}_0(t_N = 2{,}41) = Q_0 \sum_{i=1}^{N} \ln(t_i/t_{i-1}) \langle I_i \rangle,$$
$$\langle I_i \rangle = \frac{1}{2}[I(t_i) - I(t_{i-1})]. \quad (26)$$

The function $Q_0^{-1}\tilde{I}_0(t)c_0^{-1}\cdot 10^5$ is shown in Fig. 2.

Figure 3 (curves 1 and 2, respectively) show the graphs of the velocity of the vortex and the path it travels during the time interval δt based on Eq. (1)

$$V_\infty(t_i) = 4437 t_i^{-3/4}; \quad L(t_i) = 17746(t_i^{1/4} - 0.9902),$$

and this data agrees very well with the experimental results in [11].

Thus, it can be concluded from this that most mass exchange of passive impurity between the TVR and environment occurs in the initial seconds of motion of the vortex (see Fig. 2 and (26)), and during subsequent motion the ability of the vortex to lose the impurity rapidly decreases. This is also clear from the fact that the difference between the concentrations far from the vortex and on its surface according to (10) and (11), $C_0 = Q_0(P_0 t)^{-3/4} c_0$, rapidly decreases with time, and eddy diffusion decays during motion of the TVR: $D^*(t) \sim t^{-1/2}$ (see (7)).

In conclusion, we note that the mathematical model presented here not only approximately determines the amount of passive impurity lost by an ellipsoidal TVR during its motion to a medium uncontaminated by suspended particles, but it also explains the process of convective mass exchange between the vortex and the medium due to the presence of a diffusion boundary layer near the ellipsoidal shell of the TVR.

NOTATION

t_1, time of self-similar motion of the vortex; V_0, velocity of the vortex at the beginning of self-similar motion; R_0, radius of vortex at the beginning of self-similar motion; $\Psi = \Psi(t, \xi, \eta)$, dimensional stream function; $\psi = \psi(\xi, \eta)$, dimensionless stream function; $c = c(\xi, \eta)$, dimensionless concentration of passivie impurity; Q_0, maximum amount of passive impurity lost by the vortex during the entire time of self-similar motion.

LITERATURE CITED

1. M. A. Lavrent'ev and B. V. Shabat, Problems of Hydrodynamics and Their Mathematical Models [in Russian], Nauka, Moscow (1977).
2. A. T. Onufriev, "Theory of motion of a ring under the influence of gravity. Rise of a cloud from a nuclear explosion," Zh. Prikl. Mekh. Tekh. Fiz., No. 2, 3-15 (1967).
3. T. Maxworthy, "Some experimental studies of vortex rings," J. Fluid Mech., 81, Pt. 3, 465-495 (1977).
4. A. A. Buzukov "Features of the formation and motion of circular vortices in water," Zh. Prikl. Mekh. Tekh. Fiz., No. 2, 153-160 (1971).
5. A. Sommerfeld, Mechanics of Deformable Media [Russian translation], IL, Moscow (1954).
6. J. Okabe and S. Inoue, "The generation of vortex rings," Rep. Res. Inst. Appl. Mech., Kynshu Univ., 8, No. 32, 91-101 (1960).
7. V. F. Tarasov, "Evaluation of certain parameters of a turbulent vortex ring," Tr. Inst. Gidrodin. Sib. Otd. Akad. Nauk SSSR Dinam, Spl. Sredy, 14, Novosibirsk 120-127 (1973).
8. J. M. Richards, "Puff motions in unstratified surroundings," J. Fluid Mech., 21, Pt. 1, 97-106 (1965).
9. A. A. Lugovtsov, B. A. Lugovtsov, and V. F. Tarasov, "Motion of a turbulent vortex ring," Tr. Inst. Gidrodin. Sib. Otd. Akad. Nauk SSSR Dinam, Spl. Sredy, 3, Novosibirsk 50-60 (1969).
10. B. A. Lugovtsov, "Motion of a turbulent vortex ring and its transport of a passive impurity," in: Some Problems of Mathematics and Mechanics [in Russian], Nauka, Leningrad (1970), pp. 182-189.
11. V. F. Tarasov, "Experimental investigations of turbulent vortex rings," Physical Mathematical Sciences, Candidate's Dissertation, Novosibirsk (1975).
12. G. A. Korn and T. M. Korn, Mathematical Handbook for Scientists and Engineers, McGraw-Hill (1967).

13. A. T. Onufriev and S. A. Kristianovich, "Features of turbulent motion in a vortex ring," Dokl. Akad. Nauk SSSR, 229, No. 1, 42-44 (1976).
14. A. T. Onufriev, "Phenomenological models of turbulence," in: Aerodynamics and Physical Kinetics. Tr. Inst. Teor. Prikl. Mekh. Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1977), pp. 43-66.
15. L. A. Vulis and V. P. Kashkarov, Theory of a Jet of a Viscous Fluid [in Russian], Nauka, Moscow (1965).
16. V. S. Krylov, "Diffusion boundary layer on the surface of a moving drop in the presence of a bulk chemical reaction," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1, 146-149 (1967).
17. V. G. Lévich, Physicochemical Hydrodynamics, Prentice-Hall (1972).
18. R. Mises, "Bermerkungen zur Hydrodynamik," Z. Angew. Math. Mech., 7, 425-431 (1927).
19. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian], Nauka, Moscow.
20. D. W. Moore, "The velocity of rise of distorted gas bubbles in a liquid of small velocity," J. Fluid Mech., 23, Pt. 4, 749-766 (1965).

MODIFIED BOUNDARY CONDITIONS FOR TWO-DIMENSIONAL GASDYNAMIC CALCULATIONS IN REGIONS OF ARBITRARY SHAPE WITH MOVING BOUNDARIES PRESENT

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Boundary conditions enabling one to improve the accuracy, convergence, and economy on numerical calculations are discussed.

Numerical calculations of gasdynamic flows in regions with arbitrary curved boundaries are greatly complicated by the difficulties of constructing the finite-difference grid (coordinate system) and approximating the boundary conditions. Because of this, much interest has recently been devoted to the investigation of ways of generating coordinate systems, accomplished, e.g., using conformal and quasiconformal transformations, elliptic equations, and algebraic transformations [1, 2]. Several ways of automating the distribution of the coordinate lines and monitoring them have been determined and a theoretical study of the errors introduced into the solution by arbitrary coordinate systems has begun. Nevertheless, the construction of a "good" coordinate system in regions of arbitrary shape where the boundary conditions are easily assigned is still a difficult problem of independent importance. Therefore, the search for ways of using simpler procedures to describe curved boundaries and assign boundary conditions is timely.

Below we consider a method of calculating boundary cells obtained by superposing an irregular orthogonal grid onto boundaries of arbitrary shape, already proposed in the period of the first computer calculations, according to [3]. Detailed information about this so-called method of fractional cells is contained in [4], where the necessary calculating equations are given. Work is known in which modified boundary conditions were introduced within the framework of the method of fractional cells. Thus, in [5] a moving undeformed boundary, a piston, is introduced along one of the coordinate axes, and the number of types of fractional cells is reduced to two using an irregular orthogonal grid. A more universal method of calculating curved boundaries moving arbitrarily over a grid was proposed in [6]. In this case conservative equations for boundary purposes and fractional cells are used in [5, 6]. In [5, 7] the condition of nonpenetration at the fixed curved boundaries is supplemented by the condition of stream slippage, realized through reorganization of the velocity vector in the fractional cells.

The aim of the present work is to clarify the role of the boundary conditions at curved

V. I. Lenin Belorussian State University, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 50, No. 1, pp. 40-48, January, 1986. Original article submitted September 24, 1984.

 0022-0841/86/5001-0030$12.50